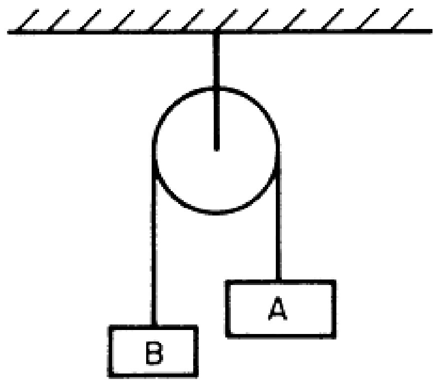
balance moon stone



A space traveler about to leave for the moon has a spring balance and a 1.0 kg mass A, which when hung on the balance on the Earth gives the reading of 9.8 newtons. Arriving at the moon at a place where the acceleration of gravity is not known exactly but has a value of about 1/6 the acceleration of gravity at the Earth's surface, he picks up a stone B which gives a reading of 9.8 newtons when weighed on the spring balance. He then hangs A and B over a pulley as shown in the figure and observes that B falls with an acceleration of 1.2 m s^{-2}. What is the mass of stone B?

Solution by Gert Hamacher

We know the gravitational acceleration on Earth *g*, and the acceleration of the masses on the moon *a*:

$$g = 9.81 \text{ m s}^{-2}$$
$$a = 1.2 \text{ m s}^{-2} \qquad (1)$$

The balance/scale registers the same weight for the stone m_B on the moon as for the mass m_A on Earth, so calling the gravitational acceleration on the moon g_M,

$$m_A \cdot g = m_B \cdot g_M ,$$
$$g_M = g \cdot \frac{m_A}{m_B}. \qquad (2)$$

Applying Newton's 2nd law $F = ma$,

$$m_B \cdot g_M - m_A \cdot g_M = (m_A + m_B) \cdot a . \qquad (3)$$

Substituting for g_M from (2) into (3) and solving for m_B,

$$m_B = \frac{(g-a) \pm \sqrt{(g-a)^2 + 4a \cdot g}}{2a / m_A} . \qquad (4)$$

Using $m_A = 1.0$ kg and the accelerations given in (1), the solution to (4) is found to be

$$m_B = 5.75 \text{ kg or } 1.42 \text{ kg}. \qquad (5)$$

These two solutions correspond to two different gravitational accelerations on the moon. However, it is given that $g_M/g \approx 1/6$, so by (2) the ratio m_A/m_B must be about $1/6$. Therefore the correct value for m_B is, within the accuracy of the scale, 5.8 kg.