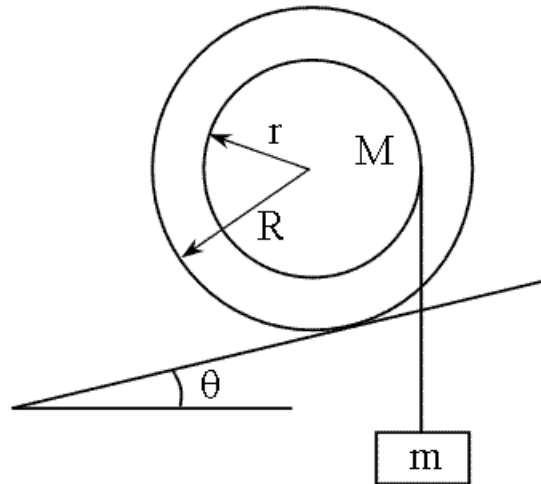


bobbin on incline



A bobbin of $M = 3$ kg consists of a central cylinder of radius $r = 5$ cm and two end plates of radius $R = 6$ cm. It is placed on a slotted incline on which it will roll but not slip, and a mass $m = 4.5$ kg is suspended from a cord wound around the bobbin. It is observed that the system is in static equilibrium. What is the angle of tilt θ of the incline?

Solution by David Peterson

The net component of force on a static body along any axis must be 0. Thus if F is the contact force exerted by the incline on the bobbin, W_b is the weight of the bobbin, and W_m is the weight of the hanging mass, then (with y vertical and x horizontal)

$$F_y - W_b - W_m = F_y - 3g - 4.5g = 0,$$
$$\therefore F_y = 7.5g,$$

and

$$F_x = 0.$$

In addition, the net torque on a static body about any axis must be 0. Consider the torque about the bobbin's axis of symmetry (i.e. through its center of mass, perpendicular to the plane of the figure): the horizontal distance from the axis to the vertical contact force of the incline is $6 \sin \theta$, so the torque from the contact force is $(6 \sin \theta) F_y = 45g \sin \theta$; the horizontal distance from the axis to the vertical force of the hanging mass is 5 cm, so the *opposing* torque from the hanging mass is $5W_m = 22.5g$. Thus,

$$45g \sin \theta - 22.5g = 0,$$
$$\sin \theta = 0.5,$$
$$\therefore \theta = 30^\circ.$$