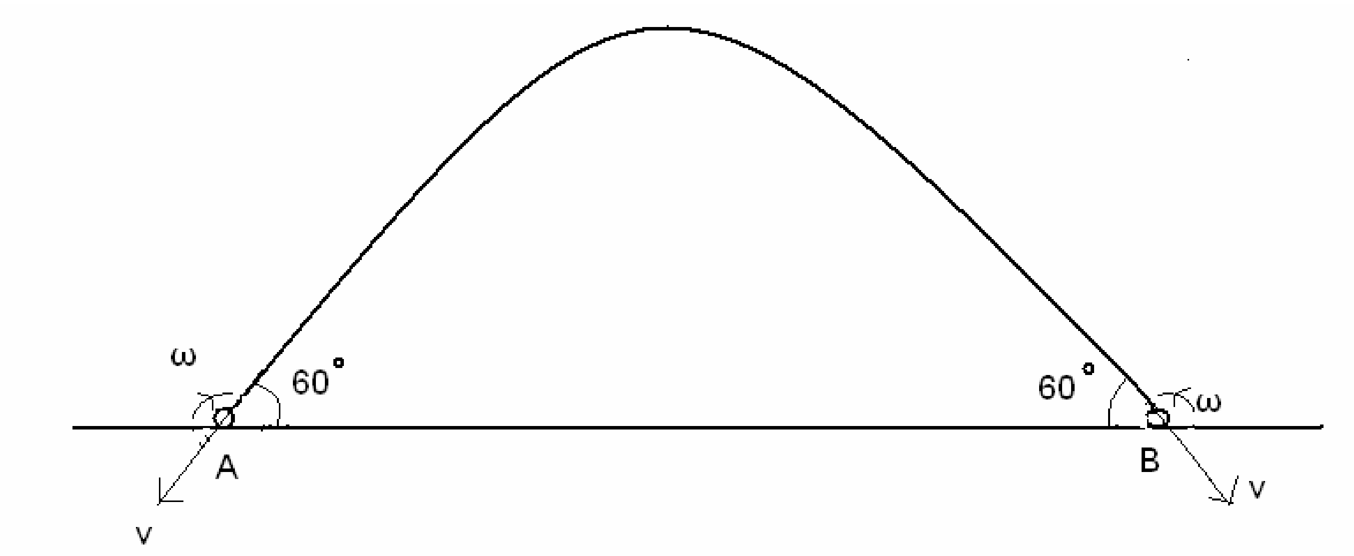
Bouncing Ball

This problem was contributed to the Feynman Lectures Website by Sukumar Chandra.



A small solid rubber ball of radius r is thrown against a rough horizontal floor such that its velocity just before striking the floor at A is v making an angle of 60^0 with the horizontal and also has a back spin of angular velocity ω. It is observed that the ball bounces from A to B, then from B to A, then from A to B, etc. Assuming perfectly elastic impact determine

a) the required magnitude of ω of the back spin in terms of v and r.

b) the minimum magnitude of co-efficient of static friction, $\mu_{s\ min}$, to enable this motion.

Sukumar Chandra's Solution

At A, the ball strikes the floor with linear velocity whose horizontal component is leftward v $\cos 60^0$ or v/2 and vertical component is downward v $\sin 60^0$ or √3v/2. Its angular velocity is clockwise ω. In order that the ball executes the repeated to and fro motion it must bounce back with linear velocity whose horizontal component is rightward v/2, vertical component is upward √3v/2 and angular velocity anti-clockwise ω. Hence during the small duration Δt the ball, of mass m (say), is in contact with floor, its

1) change in linear horizontal momentum is rightward mv. Hence, $f_s\Delta t = mv$, where f_s is the average static impulsive force of friction that comes in to play horizontally rightward during contact.
2) change in linear vertical momentum is upward √3mv. Hence, $N\ \Delta t = \sqrt{3}mv$, where N is the average impulsive normal reaction force that comes into play during the contact.
3) change in angular momentum is 2Iω in anti-clockwise direction, where I ($= 2mr^2/5$) is the moment of inertia of the solid spherical ball about its axis of rotation through its center.

a) As torque impulse = change in angular momentum, so $f_s\Delta t \times r = 2I\omega$, i.e. $mvr = 2(2mr^2/5)\ \omega$,

Hence $\omega = 5v/4r$.

b) Also the limiting value of static force of friction is μ_s times the normal reaction force, i.e. $f_s \le \mu_s N$

$\Rightarrow f_s\Delta t \le \mu_s N\Delta t$

$\Rightarrow mv \le \mu_s\sqrt{3}mv$

$\Rightarrow \mu_s \ge 1/\sqrt{3}$

$\Rightarrow \mu_{s\ min.} = 1/\sqrt{3} \approx 0.58.$