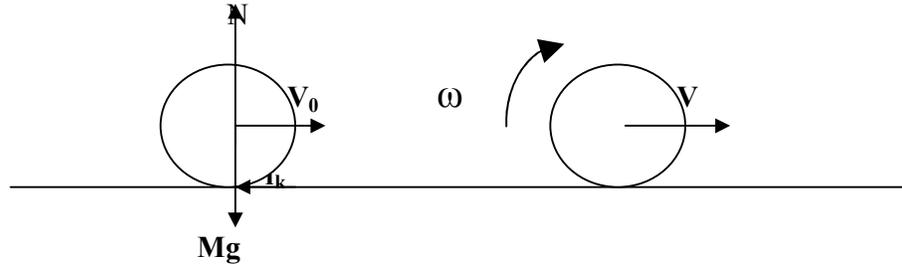


Bowling Ball Rolling

A uniform bowling ball of radius R and mass M is initially launched so that it is sliding with speed V_0 without rolling on an alley with a coefficient of friction μ . How far does the ball go before it starts rolling without slipping, and what is then its speed?

Sukumar Chandra's Solution (using kinematics)



When the ball is slipping, the forces acting on it are:

- 1) Weight Mg , acting vertically downward through the centre,
- 2) Normal reaction N , vertically upward through the point of contact
- 3) Forces of kinetic friction, $f_k = \mu N$, horizontally through the point of contact and opposite to the direction of motion.

The only horizontal force f_k produces horizontal acceleration, $a = \text{force/mass} = \mu N / M = \mu g$ [as $N = Mg$]. This acceleration is directed opposite to the direction of motion of the ball and hence its velocity V at any instant t is given by:

$$V = V_0 - \mu g t. \quad (1)$$

Further, the only force that produces a torque about the centre is f_k . This torque is of magnitude $f_k R$, acting in anticlockwise direction producing an anticlockwise angular acceleration, α , of the ball about its center given as $f_k R = I_{cm} \alpha$, or $\mu MgR = (2/5)MR^2 \alpha$, or $\alpha = 5 \mu g / 2R$. This angular acceleration sets the ball rotating with increasing angular velocity in anticlockwise direction whose magnitude ω , at any instant t , is given by

$$\omega = \alpha t. \quad (2)$$

When the ball starts pure rolling,

$$V = \omega R \quad (3)$$

is satisfied. Hence using equations (1), (2) and (3) we get, $V_0 - \mu g t = \alpha t R = 5 \mu g t / 2$, or $\mu g t = 2V_0 / 7$. Putting this value back in equation (1), we get $V = 5V_0 / 7$.

Again, using the equation of kinematics, $v^2 = u^2 + 2aD$, we get $(5V_0/7)^2 = V_0^2 - 2\mu g D$ or, $D = 12V_0^2 / 49\mu g$.