Solution to the "bug on band" problem

Adrian Cozac

20 June 2018

Let $x(t)$ be the bug position with respect to a fixed reference frame and $L(t)$ be the band length at $t > 0$. Introduce the dimensionless quantity $\delta = x/L \leq 1$, which represents a sort of normalized position of the bug on the band. On denoting by V the constant speed of the band free end (so that $\dot{L} = V$) and by v the bug speed with respect to the band, the time derivative $\dot{\delta}$ turns out to be

$$\dot{\delta} = \frac{\dot{x}}{L} - \frac{x}{L^2}\,\dot{L} = \frac{\dot{x} - \delta V}{L} = \frac{v}{L}\,, \tag{1}$$

where in the last passage the fact that the band speed at the normalized position $\delta(t)$ is just $\delta(t)V$ has been used.

The bug will reach the free end of the band after a time T such that $\delta(T) = 1$. On taking into account that $\delta(0) = 0$ and on denoting by L_0 the initial band length, so that $L(t) = L_0 + Vt$, straightforward integration of Eq. (1) gives at once

$$1 = \int_0^T \frac{v\,\mathrm{d}t}{L_0 + Vt} = \frac{v}{V}\log\left(1 + \frac{VT}{L_0}\right) \Longrightarrow \boxed{T = \frac{L_0}{V}\left[\exp\left(\frac{V}{v}\right) - 1\right]}$$

Finally, on letting $V = 1$ m/s, $v = 0.001$ cm/s, and $L_0 = 1$ m, the travel time turns out to be approximately $\exp(100000) \simeq 3 \times 10^{43429}$ seconds!