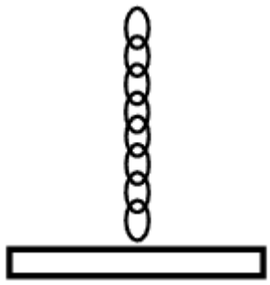
falling chain

(submitted by Ilkka Mäkinen)



A chain of length L cm and mass M kg is suspended vertically by one end with the other end immediately above a scale. The chain is released and falls. At the instant the entire chain has fallen onto the scale what does the scale read?

Solution by Sukumar Chandra:

As the chain is falling freely under gravity so at any instant t after the chain is released, when its free end goes down by x, the speed v of the chain is given as $v = \sqrt{2gx}$.

In further small interval of time Δt, a small length of chain Δx of mass (Δx M) / L strikes the scale with a speed v and comes to rest. This results in a loss of momentum of the chain of magnitude (ΔxMv) / L. As this loss takes place in time Δt, so the rate of loss of momentum of the chain is (ΔxMv) / (LΔt). Thus the scale exerts an upward force of (ΔxMv) / (LΔt) or Mv^2/ L (as $\Delta x/\Delta t = v$) on the chain which means the chain also exerts equal and opposite force on scale. Also at this instant the x length of the chain exerts its own weight too, Mgx / L on the scale. Hence the total force the chain exerts on scale when it comes down a length x is (Mv^2/ L + Mgx / L) or 3Mgx / L as $v = \sqrt{2gx}$. In other words when the chain falls by a length x, the scale reads 3Mx / L.

So when the chain has fallen a length L, the scale reads 3M.