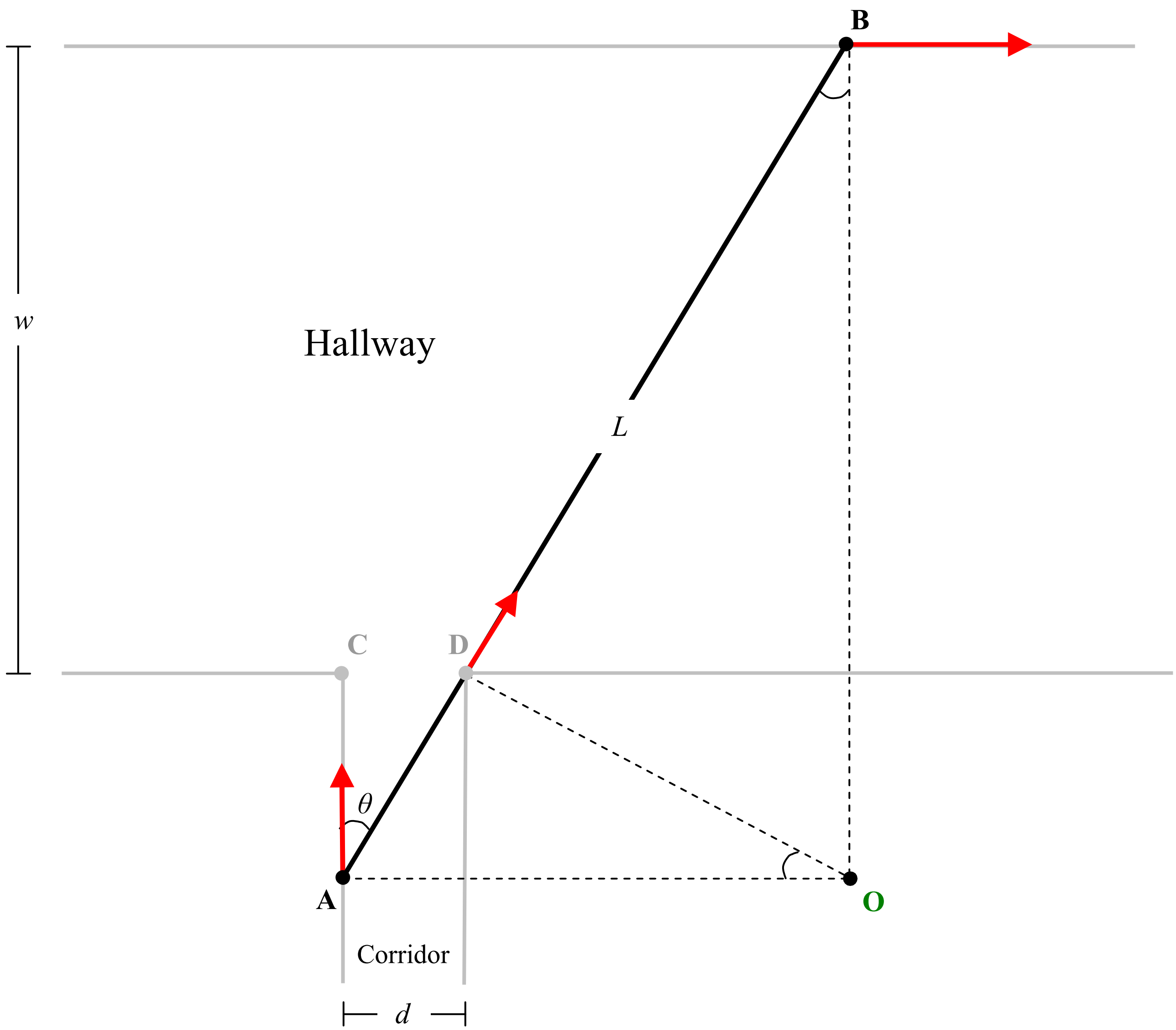
hallway pole

This problem comes from a calculus book by Lipman Bers:

You are an architect. Your client, living in Flatland, wants a building designed with a long 3 foot wide corridor which opens into a larger hallway. You must design the hallway for the minimum width w that will allow the inhabitants to move a 24 foot-long pole down the corridor and turn it into the hallway. The corridor is perpendicular to the hallway. Since this is Flatland, the pole cannot be tilted up. The pole is rigid. How wide must the hallway be?

Sergey Krotov's Solution

Refer to the following figure:



When considering the motion of a long thin pole ($\overline{\mathbf{AB}}$) we will follow the basics of planar rigid-body kinematics; All pole points "slide" on the floor. At the moment of the most distant position of the right end of the pole (point **B**) from the line of exit from the corridor ($\overline{\mathbf{CD}}$), the point **B** velocity vector is parallel to the hallway, the point **A** velocity at the other end of the pole is parallel to the corridor, while the velocity of the pole point touching corner **D** is parallel to the pole. The instantaneous rotational center of the pole – point **O** – is located at the intersection of the lines orthogonal to the velocity vectors through their respective pole points. Analyzing the sides length relations for the three similar right triangles **ACD**, **ADO** and **ABO** we find

$$d = L\sin^3\theta\,, \tag{1}$$

and

$$w = L\cos\theta - \frac{d}{\tan\theta}\,, \tag{2}$$

from which we derive the very elegant dependence,

$$w^{2/3} = L^{2/3} - d^{2/3}\,, \tag{3}$$

or

$$w = L\left[1 - \left(\frac{d}{L}\right)^{\frac{2}{3}}\right]^{\frac{3}{2}}. \tag{4}$$

Thus we find with $L = 24$ and $d = 3$ that the width of the hallway must be at least $w = 9\sqrt{3}$.