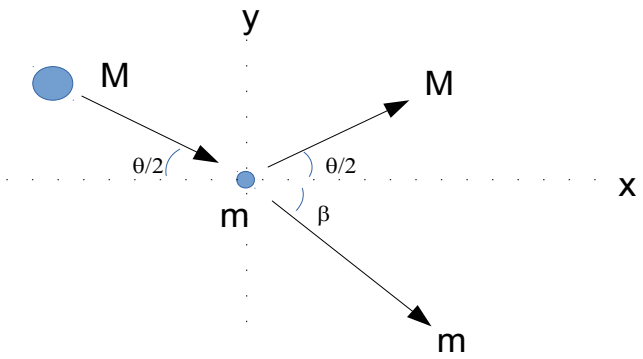
maximum angle of deflection

A moving particle of mass M collides perfectly elastically with a stationary particle of mass m < M. Find the maximum possible angle through which the incident particle can be deflected.

Solution by Julien Clément-Cottuz:

The angle of deflection θ is the angle of which the M particle is deviated. I choose (in order to simplify the trigonometric calculus) the (x,y) axes so that the angle between x and the M direction is θ/2.



v is the velocity before the collision and, v_1 and v_2 are the velocities of M and m after the collision.

Conservation of energy:

$$E=\frac{1}{2}M\,v^2=\frac{1}{2}M\,v_1^2+\frac{1}{2}m v_2^2 \quad (1)$$

Conservation of momentum:

$$P_x=M\,v\cos(\theta/2)=M\,v_1\cos(\theta/2)+m\,v_2\cos(\beta) \quad (2)$$

$$P_y=-M\,v\sin(\theta/2)=M\,v_1\sin(\theta/2)-m v_2\sin(\beta) \quad (3)$$

From (2) and (3) you get two formulae for v2 and substitute them into (1):

$$M(v^2-v_1^2)=m\,v_2^2=m\left[\frac{M\cos(\theta/2)(v-v_1)}{m\cos(\beta)}\right]\left[\frac{M\sin(\theta/2)(v+v_1)}{m\sin(\beta)}\right]$$

Simplifying:

$$1=\frac{M}{m}\frac{\cos(\theta/2)\sin(\theta/2)}{\cos(\beta)\sin(\beta)}=\frac{M}{m}\frac{\sin(\theta)}{\sin(2\beta)}$$

The deflection angle is maximum when sin(2β)=1 (so β=π/4), and sin(θ)=m/M.