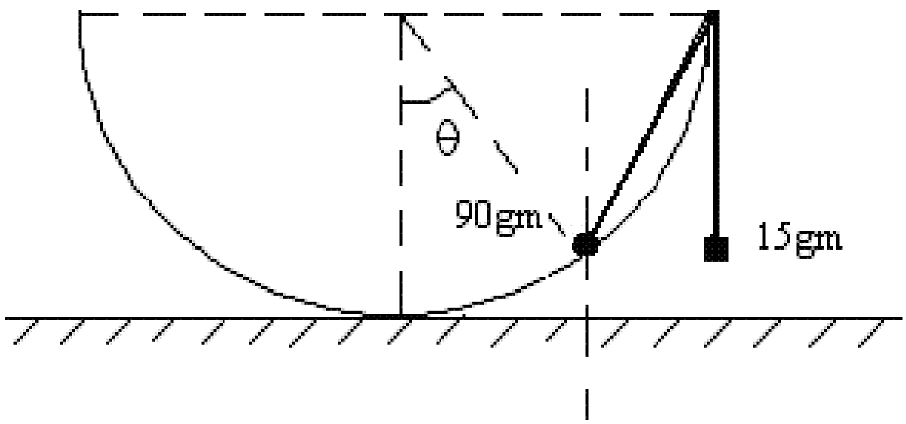
particle in bowl

This problem was contributed to the Feynman Lectures Website by Sukumar Chandra.



A hemispherical smooth bowl is held fixed with its axis vertical. A particle of mass 90gm. attached to one end of a light inextensible string is placed inside the bowl. The string passes over the frictionless rim of the bowl and is attached to another 15gm. mass hanging freely in air at its other end. In equilibrium position of the system of particles find the angle θ subtended by the radius through the 90gm. particle with the vertical.

Sukumar Chandra's Solution

Force diagram of the two particles in equilibrium is shown in Fig. a . Equilibrium of 15gm-wt implies that tension in the string, T = 15gm-wt.

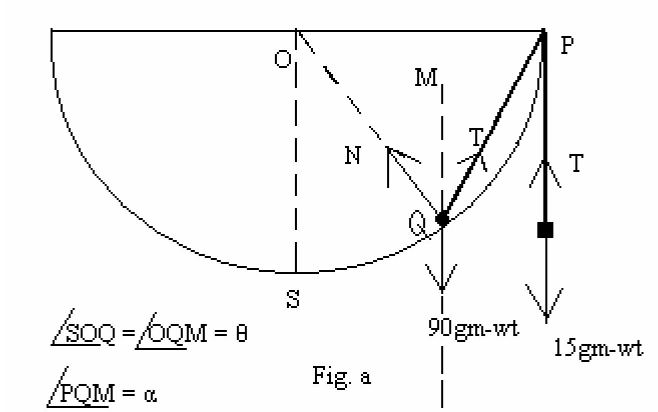


Fig. a

In triangle OQP, OQ = OP (both being equal to the radius of the bowl), so $\angle OPQ = \angle OQP = \theta + \alpha$. Also, $\angle QOP = 90° - \theta$. As the sum of the angles in a triangle is 180º, in triangle OQP $2(\theta+\alpha) + (90º - \theta) = 180º$, or

$\alpha = 45^o - \theta/2$. (1)

The forces acting on 90gm particle are: 1) tension T acting along the length of the string, 2) weight 90gm-wt, vertically downward, 3) normal reaction N along the radius of the bowl and towards its centre. These three forces keep the particle in equilibrium and hence their resultant is zero. If drawn in order these three forces should form a close triangle as shown in Fig. b. Noting that $\angle ABC = 180 - (\theta + \alpha)$, $\sin 180 - (\theta + \alpha) = \sin(\theta + \alpha)$, and applying the sine rule to this triangle we get,

$$\frac{15}{\sin\theta} = \frac{90}{\sin(\alpha + \theta)}$$

$$\Rightarrow 6\sin\theta = \sin(45^0 + \frac{\theta}{2}), \text{ (by Eq.1)}$$

$$\Rightarrow 6\sin\theta = \sin 45^0 \cos\frac{\theta}{2} + \cos 45^0 \sin\frac{\theta}{2}$$

$$\Rightarrow 6\sqrt{2}\sin\theta = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}, \text{ (as } \sin 45^0 = \cos 45^0 = \frac{1}{\sqrt{2}})$$

$$\Rightarrow 72\sin^2\theta = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}, \text{ (squaring)}$$

$$\Rightarrow 72\sin^2\theta = 1 + \sin\theta$$

$$\Rightarrow 72\sin^2\theta - \sin\theta - 1 = 0$$

$$\Rightarrow (8\sin\theta - 1)(9\sin\theta + 1) = 0$$

$$\Rightarrow \sin\theta = \frac{1}{8} \, or - \frac{1}{9},$$

as $0 \le \theta \le 90^0$, so $\sin\theta \ne -\frac{1}{9}$.

Hence $\sin\theta = \frac{1}{8}$,

or $\theta = \sin^{-1}\frac{1}{8} \approx 7.2^0$

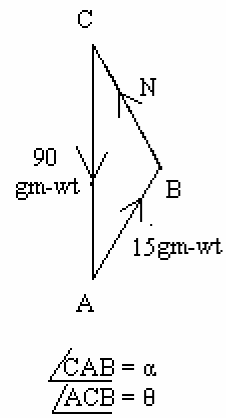


Fig. b