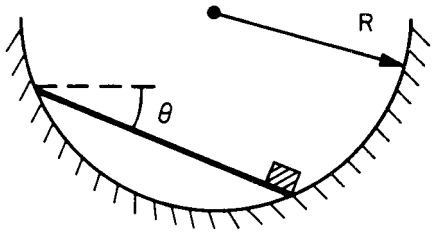
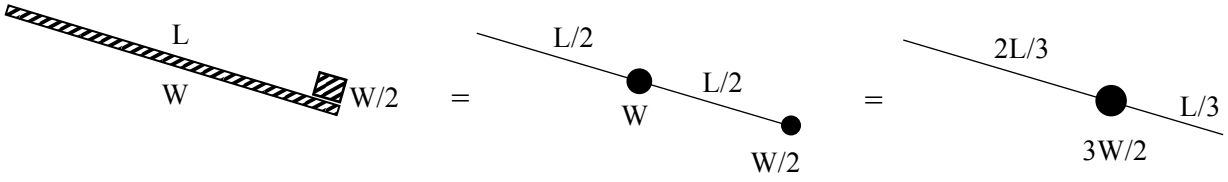
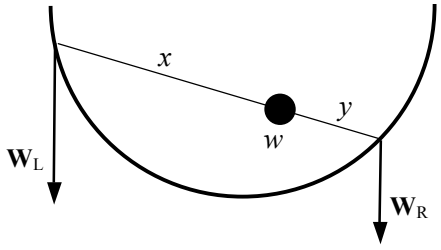
plank weight trough



A plank of weight W and length $\sqrt{3}R$ lies in a smooth circular trough of radius R. At one end of the plank is a weight W/2. Calculate the angle θ at which the plank lies when it is in equilibrium.

Michael A. Gottlieb's Solution *(notes)*

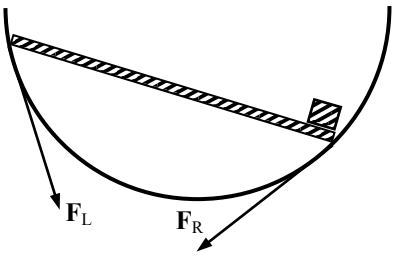


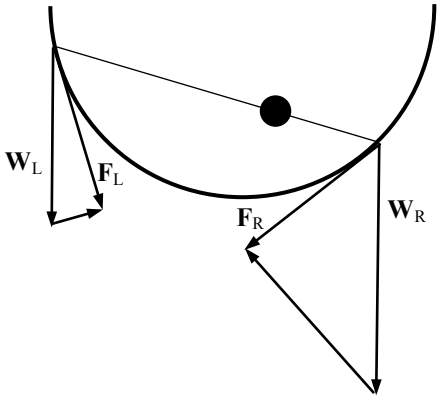


$$W_L = \frac{L/3}{L}(3W/2) \qquad W_R = \frac{2L/3}{L}(3W/2)$$

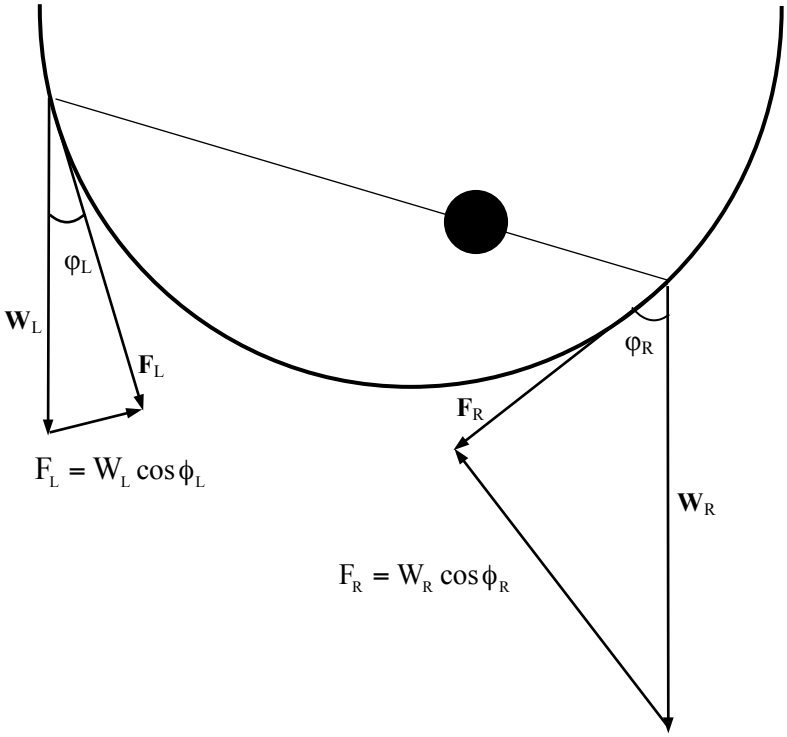
$$W_L = W/2 \qquad W_R = W$$

$$W_L = \frac{y}{x+y}w \qquad W_R = \frac{x}{x+y}w$$





$F_L = F_R$ at equilibrium



$F_L = W_L \cos\phi_L$

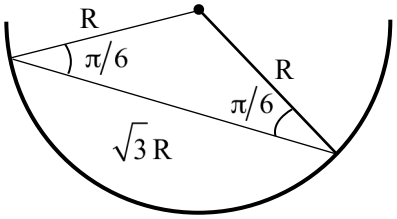
$F_R = W_R \cos\phi_R$

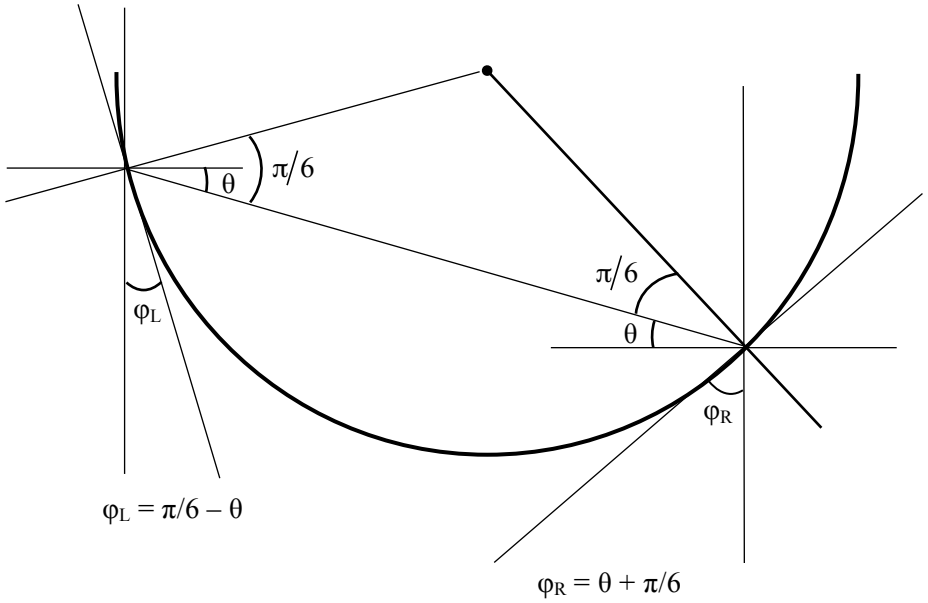
$$F_L = F_R$$

$$W_L \cos\phi_L = W_R \cos\phi_R$$

$$\left(W/2\right)\cos\phi_L = W\cos\phi_R$$

$$\therefore \cos\phi_L = 2\cos\phi_R$$





$$\cos\left(\frac{\pi}{6} - \theta\right) = 2\cos\left(\theta + \frac{\pi}{6}\right)$$

$$\therefore \theta = \frac{\pi}{6}$$

$\varphi_L = \pi/6 - \theta$

$\varphi_R = \theta + \pi/6$