rocket vs. jet

On the long horizontal test track at Edwards AFB, both rocket and jet motors can be tested. On a certain day, a rocket motor, started from rest, accelerated constantly until its fuel was exhausted, after which it ran at constant speed. It was observed that this exhaustion of fuel took place as the rocket passed the midpoint of the measured test distance. The jet motor was started from rest down the track, with a constant acceleration for the entire distance. It was observed that both rocket and jet motors covered the test distance in exactly the same time. What was the ratio of acceleration of the jet motor of that of the rocket motor?

Solution by Rudy Arthur:

Using the equations,

$$s = ut + \frac{1}{2}at^2, \qquad (1)$$

$$v^2 = u^2 + 2as \qquad (2)$$

(with u = initial velocity, v = final velocity, s = distance, t = time taken, a = acceleration):

Jet: $$s = \frac{1}{2}a_J t^2 \qquad (3)$$

Rocket: 1st part (acceleration)

$$s_1 = \frac{1}{2}a_R t_1^2. \qquad (4)$$

$$v = a_R t_1. \qquad (5)$$

$$v^2 = 2a_R s_1. \qquad (6)$$

2nd part (unaccelerated)

$$s_2 = vt_2. \qquad (7)$$

$$s_2 = a_R t_1 t_2. \qquad (8)$$

General: $$t_1 + t_2 = t. \qquad (9)$$

$$s_1 + s_2 = s. \qquad (10)$$

$$s_1 = s_2 = s/2. \qquad (11)$$

Substitute (9) into (3) to get

$$s = \frac{1}{2}a_J (t_1^2 + 2t_1 t_2 + t_2^2). \qquad (12)$$

Rearranging (4) gives $t_1^2 = \frac{s}{a_R}$. (13)

Square (7) and sub in (6) gives $t_2^2 = \frac{s}{4a_R}$. (14)

Twice (8) gives $2t_1 t_2 = \frac{s}{a_R}$. (15)

Putting (13), (14) and (15) together in (12) gives $\frac{a_J}{a_R} = \frac{8}{9}$.