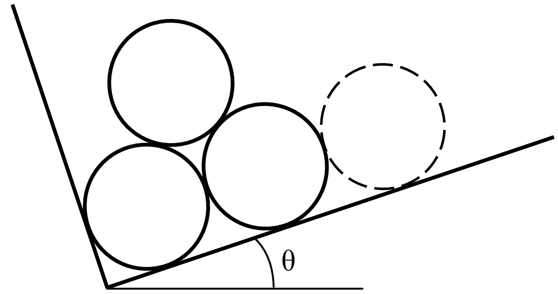
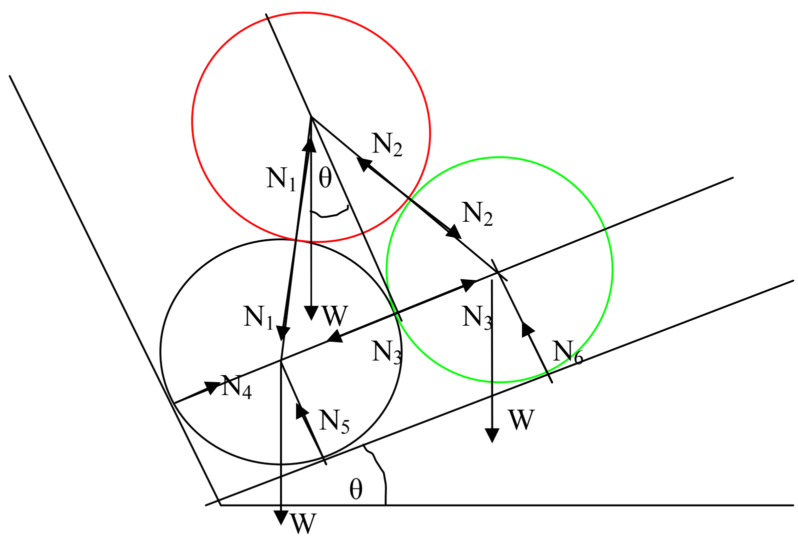
three logs



Smooth, identical logs are piled in a stake truck. The truck is forced off the highway and comes to rest on an even keel lengthwise but with the bed at an angle θ with the horizontal. As the truck is unloaded, the removal of the log shown dotted leaves the remaining three in a condition where they are just ready to slide, that is, if θ were any smaller, the logs would fall down. Find θ.

Solution by Sukumar Chandra



Detailed Force Diagram of all the logs.

W = weight of each log acting vertically downward through the respective centre of masses.

N_1 = normal action - reaction forces exchanged between the red and black log, each actig along the line joining their centres and directed towards the respective centre. N_2, N_3 are the similar action – reaction pairs.

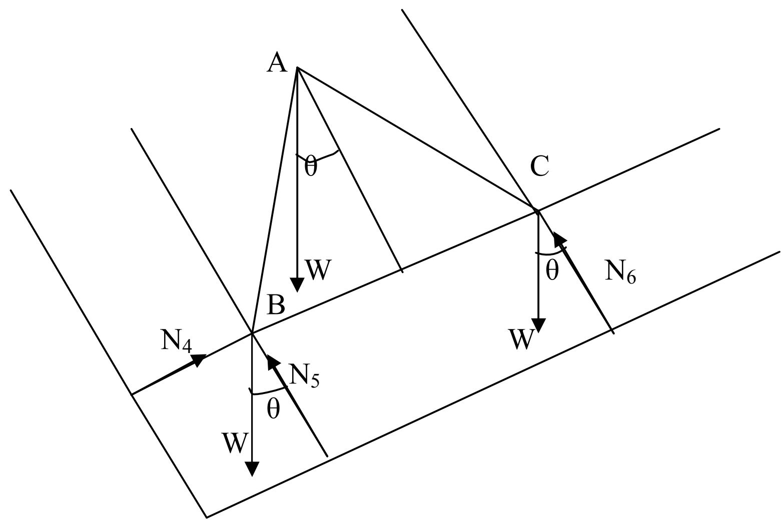
N_4 = normal reaction force exerted on the black log by the wall passing through their point of contact and directed towards the centre of the black log. Its line of action is same as that of N_3.

N_5 = normal reaction force exerted by the floor on the black log, passing through their point of contact and directed towards the centre of the black log. N_6 is the similar force on green log.

External forces on three logs as system:

1) Weight of each log, W.
2) N_4, N_5, N_6.

Force Diagram of the System of Three Logs



AB = BC = CA = 2R

Fig. 1

Considering the force diagram of three log as system (Fig.1), resolving forces in two mutually perpendicular direction (parallel and normal to the inclined plane), we get for translational equilibrium,

$$N_4 = 3W \sin\theta, \quad (1)$$

$$N_5 + N_6 = 3W \cos\theta. \quad (2)$$

Considering rotational equilibrium of the system, we can declare that torque produced by all the forces about any point is zero.Taking torque about point B (meeting point of N_4, N_5, W), we get

$$N_6 \times BC = W \cos\theta \times BC + W \sin(30^0 - \theta) \times AB, \quad (3)$$

or

$$N_6 = \frac{W}{2}(3 \cos\theta - \sqrt{3} \sin\theta) .$$

Hence from eq.(2), we get

$$N_5 = \frac{W}{2}(\sqrt{3} \sin\theta + 3\cos\theta).$$

Force Diagram of Individual Log as System

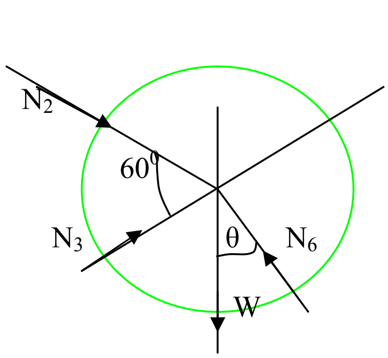


Fig. 2

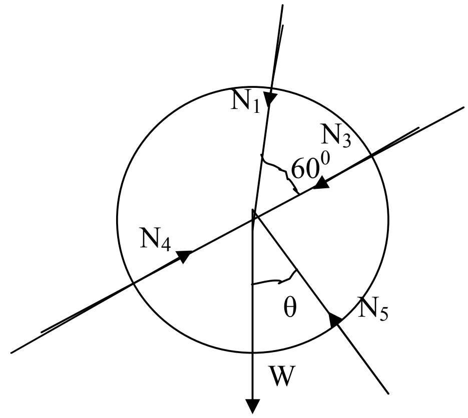


Fig. 3

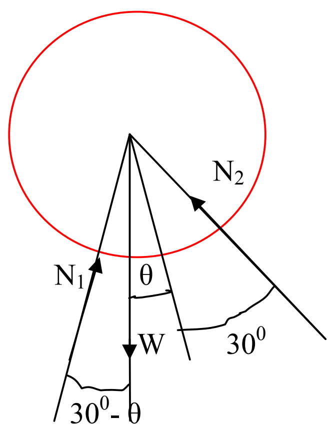


Fig. 4

If we now isolate the black log (Fig.3) from the system and study its translational equilibrium, we get

$$N_1 \, Sin60^0 + W \, Cos \, \theta - N_5 = 0 \, . \qquad (4)$$

$$N_1 \, Cos60^0 + N_3 + W \, Sin\theta - N_4 = 0. \qquad (5)$$

Solution of these two equations give us

$$N_1 = W(Sin\theta + {}^1/_{\sqrt{3}} \, Cos\theta),$$

$$N_3 = \frac{W}{2}(\, 3Sin\theta - {}^1/_{\sqrt{3}} \, Cos\theta).$$

Similar Procedure with red log (Fig. 4) yields,

$$N_1 \, Cos30^0 + N_2 \, Cos30^0 - W \, Cos\theta = 0 \qquad (6)$$

$$N_2 \, Sin30^0 + W \, Sin\theta - N_1 \, Sin30^0 = 0 \qquad (7)$$

Solving we get, $N_2 = W({}^1/_{\sqrt{3}} \, Cos\theta - Sin\theta)$.

Compiling all the normal reaction forces we get,

$N_1 = W(\text{Sin}\theta + {}^1/_{\sqrt{3}}\ \text{Cos}\theta)$,

$N_2 = W({}^1/_{\sqrt{3}}\ \text{Cos}\theta - \text{Sin}\theta)$,

$N_3 = \frac{W}{2}(\ 3\text{Sin}\theta - {}^1/_{\sqrt{3}}\ \text{Cos}\theta)$,

$N_4 = 3W\ \text{Sin}\theta$,

$N_5 = \frac{W}{2}(\sqrt{3}\ \text{Sin}\theta + 3\text{Cos}\theta)$,

$N_6 = \frac{W}{2}(3\ \text{Cos}\theta - \sqrt{3}\ \text{Sin}\theta)$.

Any of the logs will slide provided it looses contact with either of the logs, floor or the wall. *i.e.* normal reaction at the concerned point becomes non-positive or $N \leq 0$. Examinig the different values of N, we find that N_1, N_4, N_5 is always positive for $\theta > 0$. This implies that there will be no sliding at these points viz. the point of contact of the black log with the floor, wall and the red log.

$N_2 \leq 0 \Rightarrow \text{Tan}\theta \geq 1/\sqrt{3}$ or, $\theta \geq 30^0$

$N_3 \leq 0 \Rightarrow \text{Tan}\theta \leq \frac{1}{3\sqrt{3}}$, or $\theta \leq 10.9^0$ (approx.)

$N_6 \leq 0 \Rightarrow \text{Tan}\theta \geq \sqrt{3}$, or $\theta \geq 60^0$

Hence, as the angle θ increases beyond 30^0, sliding takes place between the red and green log. Beyond 60^0, sliding taes place between green log and the floor. If θ falls below 10.9^0, contact is lost between the green and black log and sliding takes place.

As the exercise suggest "if θ were any smaller, the logs would fall down", obviously our answer is $\underline{\theta \approx 10.9^0}$